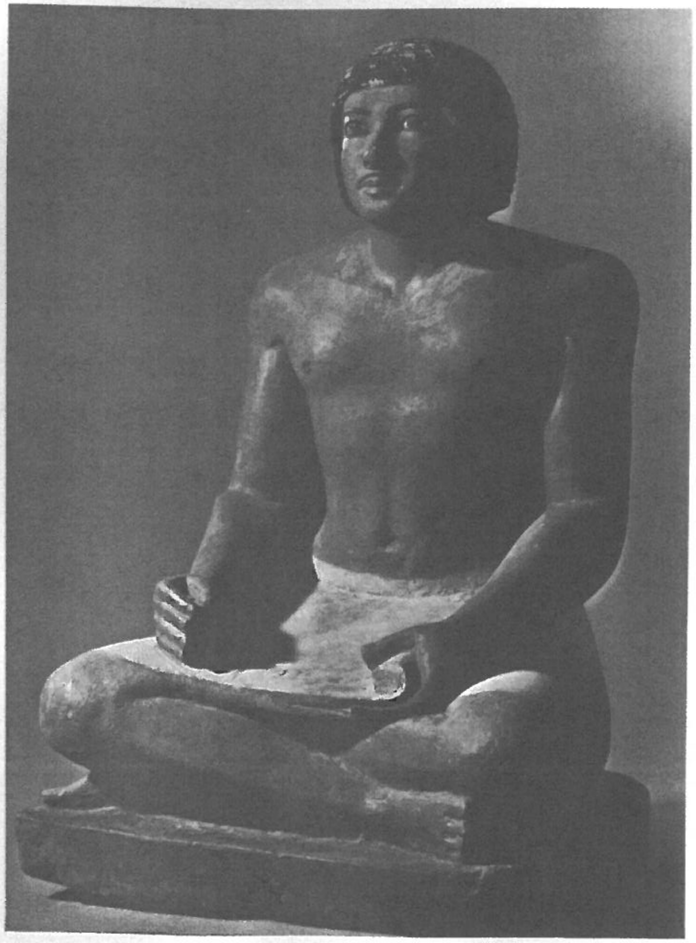

The scribe Ka-Irw-Khufu of the Fourth Dynasty, excavated at Giza. The statue is now owned by the Cairo Museum. Copyright Roger Wood, London.

MATHEMATICS IN THE TIME OF THE PHARAOHS

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Dedicated to O. Neugebauer and A. Sachs,
in friendship and gratitude

However one looks at this "round-about" method of solution, it is entirely logical and indeed elegant, whether or not the scribe arrived at it by some algebraic or symbolic thought processes, or by some other means. Whatever the true story behind it is, we can only be amazed at such an achievement in 1850 B.C., and suggest that here perhaps, as the scribe wrote it, we are looking at the very earliest example of rhetorical algebra to come to the attention of the historian of mathematics. We cannot avoid introducing the concept of alternando and dividendo in some form or other, because of the scribe's direction in line 2, "Find the excess of q over p."

13 AREAS AND VOLUMES

THE AREA OF A RECTANGLE

From the extant papyri, etc., it is clear that the scribes found the areas of rectangles by multiplying length and breadth as we do today. In Problem 49 of the RMP, the area of a rectangle of length 10 khet (1,000 cubits) and breadth 1 khet* (100 cubits) is found to be $1{,}000 \times 100 = 100{,}000$ square cubits. The area was given by the scribe as 1,000 *cubit strips*, which are rectangles, usually of land, 1 khet by 1 cubit (Figure 13.1).

Again in Problem 6 of the MMP, calculation of the area of a rectangle is used in a problem on simultaneous equations. The following text accompanied the rectangle shown on the right in Figure 13.1:

l. 1 Method of calculating a rectangle.
l. 2 If it is said to thee, a rectangle of 12 in the area [is] $\bar{2}$ $\bar{4}$ of the length
l. 3 for the breadth. Calculate $\bar{2}$ $\bar{4}$ until you get 1. Result 1 $\bar{3}$.
l. 4 Reckon with these 12, 1 $\bar{3}$ times. Result 16.
l. 5 Calculate thou its angle [square root]. Result 4 for the length.
l. 6 $\bar{2}$ $\bar{4}$ is 3 for the breadth.

FIGURE 13.1
Rectangles from the RMP and the MMP. *Left*, the 1-khet by 10-khet rectangle of Problem 49 of the RMP. Note that the scribe has made an error in copying, showing the breadth as 2 khet. The working accompanying the figure was done for a breadth of 1 khet. *Right*, a rectangle of area 12 and breadth $\bar{2}$ $\bar{4}$ of the length, from Problem 6 of the MMP. The scribe wrote his correct answers for breadth and width on the figure as shown.

* The scribe made a careless copying error, putting 2 khet for 1 khet, and he repeated it in his figure. But there are no errors in his calculations with 1 khet.

Rewriting this in modern form, we would have:
If

$$A = lb = 12 \quad \text{and} \quad b = (\bar{2}\ \bar{4})l, \qquad \text{(from l. 2)}$$

to solve for l, first substitute for b: $l \times (\bar{2}\ \bar{4})l = 12$.

Then

$$\begin{aligned} l \times l &= 12 \div \bar{2}\ \bar{4} && \text{(l. 3)} \\ &= 12 \times 1\ \bar{3} && \text{(l. 3)} \\ &= 16. && \text{(l. 4)} \end{aligned}$$

Therefore

$$l = 4 \text{ for the length,} \qquad \text{(l. 5)}$$

and

$$\bar{2}\ \bar{4} \text{ of 4 is the breadth 3.} \qquad \text{(l. 6)}$$

THE AREA OF A TRIANGLE

For the area of a triangle ancient Egyptians used the equivalent of the formula $A = \frac{1}{2}bh$. In RMP 51 the scribe shows how to find the area of a triangle of land of side* 10 khet and of base† 4 khet. The scribe took the half of 4, then multiplied 10 by 2 obtaining the area as 20 setats of land. Then in MMP 4 the same problem was stated as finding the area of a triangle of height* 10 and base† 4. As stated by the scribe the method was to calculate with a half of 4 and then to reckon with 10 twice, giving an area of 20. No units such as khets or setats were mentioned.

There have been differences of scholarly opinion among philologists

FIGURE 13.2
Triangles accompanying Problem 51 of the RMP (*left*) and Problem 4 of the MMP (*right*).

* The word *meret* (or *meryet*) translated as side or height.
† The word *teper* (or *tepro*) translated as base or mouth.

regarding the precise meaning of *meret* and of *teper*. If *meret* meant the side rather than the height, the area would be in error unless the triangle were right-angled. Scribal sketches of triangles in the papyri suggest that a right-angle may have been intended, but some observers have thought that isosceles triangles with acute vertical angles were meant, and if *meret* was side the area would again be in error by a small amount. However, these differences of opinion are academic; and modern-day historians agree that perpendicular height is meant by the scribe. Thus Peet writes* that he believes "that the Egyptians had found the correct formula, half the base multiplied by the vertical height for the scalene triangle."

D. J. Struik writes,† "The area of a triangle was found as half the product of base and altitude," while Carl B. Boyer has,‡ "Problem 51 (of Ahmes) shows that the area of an isosceles triangle was found by taking half of what we would call the base and multiplying this by the altitude."

THE AREA OF A CIRCLE

In Problem 50 of the RMP, the scribe showed how to find the area of a circle. To explain his method, he assumed a circle of diameter 9 khet as a matter of arithmetical convenience and not because it is a really practical problem. A khet was 100 royal cubits (approximately 57 yards); a *square khet* was called a *setat* (about two-thirds of an acre). The circle the scribe refers to would have an area of over 40 acres and a circumference of nearly a mile.

FIGURE 13.3
Circles drawn by the scribe of the RMP. *Left*, from Problem 41; *right*, from Problem 50.

* T. E. Peet, "Mathematics in Ancient Egypt," *Bulletin of the John Rylands Library*, Vol. 15, No. 2 (Manchester, 1931), p. 430.
† D. J. Struik, *A Concise History of Mathematics*, Dover, New York, 1948, p. 22.
‡ Carl B. Boyer, *A History of Mathematics*, Wiley, New York, 1968, p. 18.

A circle with hieratic inscriptions (Figure 13.3) was included by the scribe in his statement of Problem 50. A translation of Problem 50 is:

Take away $\bar{9}$ of the diameter, namely, 1.
The remainder is 8.
Multiply 8 by 8.
It makes 64.
Therefore it contains 64 setat of land.

Do it thus.

1	9
$\bar{9}$	1.

The remainder is 8.

1	8
2	16
4	32
\8	\64

The area is 64 setat.

The scribe's method of finding the area of a circle can thus be restated: *Subtract from the diameter its one-ninth part, and square the remainder. This is its area.* We ask ourselves how close this is to the true value, and how did the scribe arrive at his formula? If we use the modern value for π, a circle of diameter 9 khet would have an area of 63.6174 setat, so that the Egyptian value is in error by less than 0.6 of one percent.

FIGURE 13.4
Problem 48 of the RMP as the scribe wrote it, including his geometrical illustration.

Problem 48 of the RMP is unique among the 87 problems of the papyrus in that the usual statement of what Aʿh-mosè proposed to do was replaced by a geometrical illustration (Figure 13.4) from which, clearly, the reader is expected to deduce and to understand the nature of the problem. A. B. Chace in his translation of the RMP has written:

PROBLEM 48
Compare the area of a circle and of its circumscribing square.

The circle of diameter 9		The square of side 9	
1	8 setat	\1	9 setat
2	16 setat	2	18 setat
4	32 setat	4	36 setat
\8	64 setat	\8	72 setat
		Total	81 setat

We note that Problem 50 and the first part of Problem 48 appear to be the same, yet their expressions are by no means identical. Indeed, even the numbers as written in the hieratic are not the same. In Problem 48, the 9 is written , the special sign for 9 khet, also used for 9 hekats of grain, instead of the normal hieratic 9, written , as in Problem 50 and also in the geometrical diagrams of these problems, where in both it means 9 khet. In Problem 50 the scribe is thinking and writing ordinary numbers simply as units, whereas in Problem 48 he is thinking of setats or square khets as his units, and this is the important circumstance that gives us the clue as to the true purpose of Problem 48. We must conclude that Chace was in error when in translating he interpreted the scribe's working as meaning that the figure is supposed to represent "a circle and its circumscribing square." Chace himself must have had some doubts about the circle, even though he may have thought that it was good enough for a freehand drawing with a scribal reed pen, or fine brush, for he had only to glance at the excellent freehand circle of diameter 9 khet of Problem 50, drawn with the same hand with just two confident sweeps of the pen, to see how good a calligrapher Aʿh-mosè really was. Another circle of diameter 9 cubits in Problem 41 was equally well drawn by Aʿh-mosè (Figure 13.3), and there are other circles in Problems 42 and 43 which support us in the conclusion that the figure within the square of Problem 48 is an octagon with straight sides drawn within a previously drawn square of side 9 khet, and *not* a circle with a square circumscribing it as Chace supposed. Let us look more closely at this octagon of Figure 13.4 now that we agree that it really is one. Was Aʿh-mosè intending to inscribe a regular octagon by joining eight points on the four sides of the square? The answer is no! We have no evidence that the ancient Egyptians knew the geometrical construction for determining these points, such that by cutting off the four corners of a square the resulting figure would be an eight-sided figure with all the sides of equal length. And even if Aʿh-mosè *did* know of such a construction, he would also know that it would produce a regular octagon of area most certainly greater than that of a circle of diameter 9 units, for it would be an escribed octagon like the square itself. Of course, it would be a much closer approximation to the area of the circle than the square would be, but he would also surely see how he could find a much closer approximation to the circled area, and with a much simpler and more obvious construction. All he would need to do would be to join the adjacent *points of trisection* of the sides. That this is in fact what he did, or was aiming to do, is suggested by his careful choice of a square of side 9 units to allow of easy trisection. Such an octagon would have each pair of opposite sides equal, and in the papyrus except for the top right-hand corner this certainly appears to be the case.

Vogel (1958) came to the same conclusion when he compared our modern formula for the area of a circle, $F = \pi(d/2)^2$, with the equivalent of the Egyptian formula, $F = (8d/9)^2$,* from which one derives an Egyptian value for π of $^{256}/_{81}$, which is approximately 3.1605. Vogel then remarks, "Just how this remarkably close approximation was found, we do not know, but we can offer a suggestion on examining the diagram of RMP 48." He then refers to the diagram of Figure 13.5, which he says "... seems to represent a figure whose area approaches the area of a circle inscribed in the square."†

* K. Vogel, Vorgriechische Mathematik, Vol. 1.
† *Ibid.*, "scheint einen Kreis in Annaherung darzustellen, der einem Quadrat einbeschrieben ist."

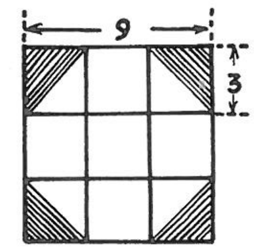

FIGURE 13.5
Vogel's diagram of the inscribed octagon of Problem 48 of the RMP.

Then, he says, the area of the octagon is equal to the original square less the two small squares made up by the four cut-off corners. "Then the area of the octagon is $(81 - 4 \times {}^{9}/_{2}) = 63$, which would correspond to a square whose side is $\sqrt{63}$, which is approximately $\sqrt{64} = 8$. Thus probably the area of a circle formula $(8d/9)^2$ might have originated."*

ALTERNATE METHOD OF OBTAINING THE FORMULA

By drawing a diagram as that shown in Figure 13.6 on a piece of papyrus, the scribe would conclude that the octagon was pretty closely equal in area to the inscribed circle because some portions of the circle are outside the octagon and some portions of the octagon are outside the circle, and mere observations by the naked eye suggest these are roughly equal. He then would sketch a square of sides representing 9 khet, trisect the sides, join the adjacent points of division, and then by drawing all the lines necessary to actually see or visualize each of the square khets or setats he can count these squares in any way he pleases to find the number of them in the octagon (see Figure 13.6).

Now the Egyptian scribes found the areas of squares and rectangles with ease. Then if the two top shaded corners of 4 $\bar{2}$ setats (or square khets) each, which add to 9 setats, were to replace the top row of 9 setats and if, similarly, the two bottom shaded corners of 9 setats were to replace the left-hand column of 9 setats, then the figure remaining

* *Ibid.*

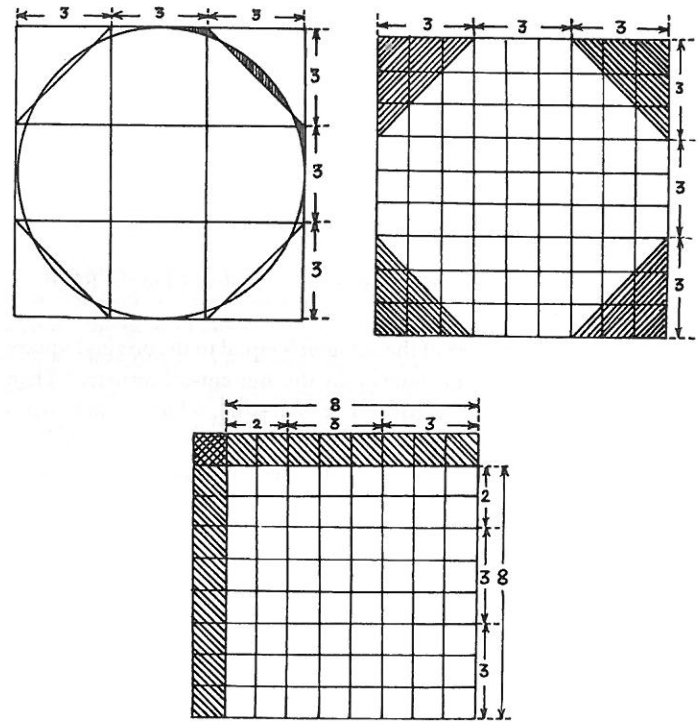

FIGURE 13.6
Diagrams for the alternate method of finding areas of circles. *Left*, the circle and the octagon; *right*, octagonal area marked out in setats or square khets; *below*, the setats from the corners now on two sides.

would be a square instead of an octagon, the area of which the scribe can easily calculate.

The scribe could now properly conclude that the area of the circle inscribed in a square of side 9 khets is very closely equal to the area of a square of side 8 khets.

Therefore, in Problem 48 the scribe finds the total number of setats in this square of eight rows of 8 setats, which he gives as the area of a circle of diameter 9 khets. Of course he certainly knows his method is not *exact*, because he has, so to speak, cut off one setat twice—the one in the top left-hand corner. But his method allows him to find a square nearly equal to a circle, so that we can, "*en caprice*," as it were, credit Aʿh-mosè with being the first authentic circle-squarer in recorded history! Problem 48 shows us that in counting the number of setats in his new square he notes that there are eight rows, each containing 8 setats, so that he multiplies 8 setats by 8, giving him 64 setats. Each step of his calculation gives the product in the special setat hieratic number signs. He does the same in his second multiplication, where he finds the number of setats in the original square to be 81 setats.

Nowhere in this problem does the scribe give the direction, "Take away thou one-ninth of the diameter," as he does in the four other Problems 41, 42, 43, and 50, for this is where he is showing how he discovered his now classical rule. And now we have to meet the critics of ancient Egyptian mathematics, e.g., Sloley,* "Mathematical knowledge in ancient Egypt was essentially practical in character, and must have developed as occasion arose in dealing with problems encountered in daily life." Then Peet, ". . . the Egyptians evolved no better means of stating a formula than that of giving three or four examples of its use, and this is hardly a tribute to the scientific nature of their mathematics."† Such statements as these are legion in histories of mathematics, and they bring to our attention a consideration of the question, What is the nature of proof?

Nowadays one tends to consider that an argument to be rigorous

* R. W. Sloley, in *The Legacy of Egypt*, S. R. K. Glanville, ed., Oxford University Press, London, 1942, p. 173.
† T. E. Peet, "Mathematics in Ancient Egypt," *Bulletin of the John Rylands Library*, Vol. 15, No. 2 (Manchester, 1931), p. 439.

must be symbolic. This is not true. A nonsymbolic argument can be quite rigorous, even when given for a particular value of the variable* (in this case it is 9). The only requirements are that the particular value be *typical* and that the generalization to any value be *immediate*. Both of these requirements are satisfied in the argument we have attributed to the scribe in deducing his method, for the value of the diameter, 9, *is* typical, and the generalization to *any* value like 12 or 15 is immediate, as it is also to a number like 20, even though it takes a little longer. The rigor of the argument is implicit in the deduction.

Peet devotes five pages to Egyptian geometry but only eight lines to the circle, even though there are five problems of the RMP dealing with its area. He writes:

> The best achievement of the Egyptians in two-dimensional geometry is undoubtedly their close approximation to the area of the circle. They squared eight-ninths of its diameter, giving $256/81r^2$, where r is the radius. Comparing this with our own πr^2 we get for the Egyptian value of π 256/81, or 3 13/81, a very close approximation to 3 1/7, which we find good enough for practical purposes. We have no idea how this result was obtained. The expression of the area as a square suggests a graphic solution.†

Peet's last sentence appears certainly to have been prophetic if the analysis based on Figure 13.6 is accepted.

THE VOLUME OF A CYLINDRICAL GRANARY

The method for finding the volume of a cylinder was, just as it is today, to first find the area of the circular base and then multiply by the height. In Problem 41 of the RMP, for a granary of diameter 9 cubits and height 10 cubits, the scribe subtracted from 9 its $\overline{9}$ part, then multiplied 8 by 8, giving 64, then multiplied by 10, to get 640 cubic cubits. This he expresses in a more convenient unit for grain by multiplying by 1 $\overline{2}$, for there are 1 $\overline{2}$ *khar* in a cubic cubit, so the volume is found to be 960 khar. This last step is analogous to a modern calculation in which the volume would be found in cubic feet, then

* I am indebted to Mr. H. Lindgren of Canberra, A.C.T., for suggesting this aspect of Problem 48.

† "Mathematics in Ancient Egypt," p. 434.

afterwards be converted to conventional bushels, or cubic feet to gallons of liquid. This is a first, easy problem, to show the method. Problem 42, which we now discuss, is much more difficult and requires some complicated arithmetic. In this problem we are required to find the contents in khar of a cylindrical granary of diameter 10 cubits and height 10 cubits.

The scribe of the RMP, by the method which he revealed in Problem 41, found the contents to be 1185 $\overline{6}$ $\overline{54}$ khar, and the arithmetic, which involves many operations in unit fractions, will well repay a careful examination. But first we look at a modern solution, using the Egyptian equivalent of π, $^{256}/_{81}$.

$$\begin{aligned} V &= \pi r^2 h \\ &= {}^{256}/_{81} \times 5^2 \times 10 \\ &= 79^1/_{81} \times 10 \\ &= 790^{10}/_{81} \text{ cubic cubits.} \end{aligned}$$

To express this in khar, add to $790^{10}/_{81}$ its half, which is $395^5/_{81}$, which gives a total of $1185^5/_{27}$ khar. Now we express $^5/_{27}$ in unit fractions, in the Egyptian tradition:

$$^5/_{27} = {}^3/_{27} + {}^2/_{27} = \overline{9} \quad \overline{18} \quad \overline{54} = \overline{6} \quad \overline{54} \text{ khar.}$$

Here is a complete statement of the scribe's solution of Problem 42:

Take away $\overline{9}$ of 10 namely 1 $\overline{9}$.
The remainder is 8 $\overline{\overline{3}}$ $\overline{6}$ $\overline{18}$.
Multiply 8 $\overline{\overline{3}}$ $\overline{6}$ $\overline{18}$ by 8 $\overline{\overline{3}}$ $\overline{6}$ $\overline{18}$.
It makes 79 $\overline{108}$ $\overline{324}$ square cubits.

Method of working out:

l. 1	1	8	$\overline{\overline{3}}$	$\overline{6}$	$\overline{18}$		
l. 2	2	17	$\overline{\overline{3}}$	$\overline{9}$			
l. 3	4	35	$\overline{2}$	$\overline{18}$			
l. 4	\ 8	\71	$\overline{9}$				
l. 5	\ $\overline{\overline{3}}$	\ 5	$\overline{\overline{3}}$	$\overline{6}$	$\overline{18}$	$\overline{27}$	
l. 6	$\overline{3}$	2	$\overline{\overline{3}}$	$\overline{6}$	$\overline{12}$	$\overline{36}$	$\overline{54}$
l. 7	\ $\overline{6}$	\ 1	$\overline{3}$	$\overline{12}$	$\overline{24}$	$\overline{72}$	$\overline{108}$
l. 8	\ $\overline{18}$	\	$\overline{3}$	$\overline{9}$	$\overline{27}$	$\overline{108}$	$\overline{324}$
l. 9	Total	79	$\overline{108}$	$\overline{324}$.			

l. 10	1	79	$\overline{108}$	$\overline{324}$		
l. 11	10	790	$\overline{18}$	$\overline{27}$	$\overline{54}$	$\overline{81}$
l. 12	$\overline{2}$	395	$\overline{36}$	$\overline{54}$	$\overline{108}$	$\overline{162}$
l. 13	Total	1185	$\overline{6}$	$\overline{54}$	[khar].	

It is left to the reader to examine and explain how the scribe:

found $\overline{9}$ of 10 to be 1 $\overline{9}$;
subtracted 1 $\overline{9}$ from 10 to give 8 $\overline{\overline{3}}$ $\overline{6}$ $\overline{18}$;
squared the number 8 $\overline{\overline{3}}$ $\overline{6}$ $\overline{18}$ to get 79 $\overline{108}$ $\overline{324}$;
multiplied this by 10 to get 790 $\overline{18}$ $\overline{27}$ $\overline{54}$ $\overline{81}$; and
multiplied this by 1 $\overline{2}$ to find the contents, 1185 $\overline{6}$ $\overline{54}$ khar.

In Problem 43, the third problem on the contents of a cylindrical granary in the RMP, the scribe proposed to show how the contents may be found directly in khar without having first to calculate the volume in cubic cubits. The scribe did this by means of a very neat and interesting transformation that, correctly interpreted, throws light on the mental processes of the Egyptian scribes. But the copyist Aʿh-mosè made two errors in his copying of the earlier work from which the RMP was derived. Because of these errors, it was some years before the problem was properly understood upon the work of Schack-Schackenburg, who correctly interpreted a similar problem in the KP.* In that problem, KP IV, 3, the scribe of the KP obtains the contents of a cylindrical granary directly in khar without first finding the volume in cubic cubits, by adding to the diameter its one-third part, squaring this number, then multiplying by two-thirds of the height. In copying a similar calculation in Problem 43 of the RMP, Aʿh-mosè gave the wrong dimensions for the silo; then he added an extra line of directions, which almost certainly was a carry-over from the two previous problems. The extra line, "take away $\overline{9}$ of the diameter," should have been *replaced* by, "add to the diameter its $\overline{3}$," and *not* been an addition to it. With these corrections made, Problem 43 should read:

A cylindrical granary of diameter 8 and height 6. What is the amount of grain that goes into it?

* F. Griffith, *The Petrie Papyri: Hieratic Papyri from Kahun and Gurob*. University College, London, 1897, pp. 15–18.

Method of working out:

	\1		\8	
	$\overline{\overline{3}}$		5	$\overline{\overline{3}}$
	\\$\overline{3}$		\2	$\overline{\overline{3}}$
Totals	1 $\overline{3}$		10	$\overline{3}$.

	1		10	$\overline{\overline{3}}$	
	\10		\106	$\overline{\overline{3}}$	
	\\$\overline{\overline{3}}$		\ 7	$\overline{9}$	
Totals	10 $\overline{\overline{3}}$		113	$\overline{\overline{3}}$	$\overline{9}$.

1	113	$\overline{\overline{3}}$	$\overline{9}$	
2	227	$\overline{2}$	$\overline{18}$	
Totals \4	\455	$\overline{9}$		khar, the answer.*

The scribe does not bother to show that 4 is two-thirds of 6 cubits. The second and third lines of doubling show a neat piece of mental reckoning:

1		113		$\overline{\overline{3}}$	$\overline{9}$	
2		226	1	$\overline{3}$($\overline{6}$	$\overline{18}$)	(Recto 2 ÷ 9)
	or,	227		$\overline{2}$	$\overline{18}$	(G rule)
\4		\454		1	$\overline{9}$	
	or,	455	$\overline{9}$.			

By taking π equal to $3^1/_7$, we find that the scribe's answer exceeds the correct value by only 1.8 percent. It is easy enough for us to determine how the scribe's transformation can be arrived at; but it is quite another matter to determine how the ancient Egyptians themselves arrived at the transformation with the much cruder methods and arithmetical tools at their disposal. Despite these handicaps their formula, checked against their standard rule, is found to be correct to within 0.56 percent, also in excess. Let us attempt to see how this may

* We can find modern counterparts to this simplification of a formula in almost any technical manual. For example, in *Shapes and Sections* (a manual of Lysaght's, a subsidiary of B. H. P., the Australian firm, Melbourne, 1937, p. 438), we find the dimensions of a cylindrical tank given in feet, the contents being found directly in imperial gallons rather than in cubic feet, by means of the formula V (in gallons) $= 5\,d^2h$.

have been done, using only those operations that were available to the scribes. The rules are as follows.

STANDARD RULE

Subtract from the diameter its one-ninth.
Multiply this number by itself.
Multiply by the height.
Multiply this number by 1 $\bar{2}$ (or add to it its half).

NEW FORM OF RULE

Add to the diameter its one-third.
Multiply this number by itself.
Multiply by two-thirds of the height.

For our own convenience we put d and h for diameter and height, and we thus have to transform

$$(d - \bar{9}d) \times (d - \bar{9}d) \times h \times (1 \quad \bar{2})$$

into

$$(d + \bar{3}d) \times (d + \bar{3}d) \times \bar{\bar{3}} \times h.$$

Now,

$(d - 9d) \times (d - 9d) \times h \times (1 \quad \bar{2})$

l. 1	$= (\bar{\bar{3}} + \bar{6} + \overline{18})d \times (\bar{\bar{3}} + \bar{6} + \overline{18})d \times h \times (1\ \bar{2})$
l. 2	$= (\bar{\bar{3}} + \bar{6} + \overline{18}) \times 1\ \bar{2} \times (\bar{\bar{3}} + \bar{6} + \overline{18}) \times (1\ \bar{2}) \times d \times d \times h \div (1\ \bar{2})$
l. 3	$= (1 + \bar{4} + \overline{12}) \times (1 + \bar{4} + \overline{12}) \times d \times d \times h \times (\bar{\bar{3}})$
l. 4	$= (1 + \bar{3})d \times (1 + \bar{3})d \times \bar{\bar{3}}h$
l. 5	$= (d + \bar{3}d) \times (d + \bar{3}d) \times \bar{\bar{3}}h.$

This is the sequence of operations that could have been made by the scribe (but not, of course, in this form), although it is quite possible that some other sequence might be found to be equally tenable. Line 1 of the sequence, $(1 - \bar{9}) = \bar{\bar{3}}\ \bar{6}\ \overline{18}$, is shown in line 1 of Problem 42 of the RMP (p. 147). In line 2 of the sequence, multiplication by an extra 1 $\bar{2}$ is balanced by division by 1 $\bar{2}$, which in line 3 becomes a

multiplication by $\bar{\bar{3}}$, the well-recognized reciprocal. And again in line 3, multiplying $\bar{\bar{3}}\ \bar{6}\ \overline{18}$ by 1 $\bar{2}$ is

\1	\ $\bar{\bar{3}}$ $\bar{6}$ $\overline{18}$	
\ $\bar{2}$	\ $\bar{3}$ $\overline{12}$ $\overline{36}$	
1 $\bar{2}$	$(\bar{\bar{3}}\ \bar{3})(\bar{6}\ \overline{12})(\overline{18}\ \overline{36})$,	
or	1 $\bar{4}$ $\overline{12}$,	(G rule)
and is line 4,	1 $\bar{3}$.	(G rule)

However the scribe may have devised the method of Problem 43, whether it was along the lines I have described or any other, there can be no doubt that, for his era, he was a mathematician of no mean ability.

In Problems 41, 42, and 43 of the RMP, the answers given here in khar are expressed in terms of hekats of grain (where 20 hekats equal 1 khar); but as the numbers are likely to be large for a granary, instead of multiplying by 20 the scribe divides by 20 and calls the answer hundreds of quadruple hekats. This is not of importance regarding the volume of cylinders, but it will come up again when tables of weights and measures are discussed.

THE DETAILS OF KAHUN IV, 3

As in other mathematical workings contained in the Kahun Papyrus, no explanation of what he proposes to do is given by the scribe. All that one reads is the drawing shown in Figure 13.7, accompanied by the following calculation:

	\ 1	\12
	$\bar{\bar{3}}$	8
	\ $\bar{3}$	\ 4
Totals	1 $\bar{3}$	16
	\ 1	\ 16
	\10	\160
	\ 5	\ 80
Totals	16	256

	\1	\ 256
	2	512
	\4	\1024
	$\backslash\overline{3}$	\ 85 $\overline{3}$
Totals	5 $\overline{3}$	1365 $\overline{3}$.

The scribe was here finding the contents of a cylindrical granary of diameter 12 (cubits) and height 8 (cubits). Now if he had followed the standard procedure illustrated by the scribe of the RMP in Problem 50, he would have taken away $\overline{9}$ of the diameter 12, squared this, and then multiplied by the height 8. This would have given him the volume 910 $\overline{6}$ $\overline{18}$ in cubic cubits. If he required the answer in khar, he would add a half of 910 $\overline{6}$ $\overline{18}$ to itself, giving 1365 $\overline{3}$ khar, since there are $1\frac{1}{2}$ khar in a cubic cubit. This is the answer the scribe has written within his freehand-drawn circle to represent the cylindrical granary.

But here the scribe of the KP has used a different technique for the volume of a cylindrical granary, by means of which the contents is determined directly in khar, and it gives evidence of considerable ingenuity on the part of the ancient Egyptian who devised it. In modern terms, it amounts to establishing the "new rule" given on p. 150. The interpretation of the scribe's method puzzled the original translator, F. L. Griffith, who wrote,

> It would seem as though the problem had been to find the contents of a circular granary, of which the height and the diameter were 12 and 8 cubits respectively; but if so the method adopted and the result are quite wrong, whether we look for the answer in cubits cubed, in khar, (= 2/3 cubits cubed), or in quadruple heqat.*

FIGURE 13.7
The circle drawn in KP IV, 3, with a translation of the accompanying hieratic.

* *The Petrie Papyri.*

With so little indication given by the scribe, it is not so surprising that the numbers 12 and 8 written outside the circle were erroneously considered by the translator to refer to the height and diameter, rather than the opposite. Thus the scribe shows the working according to the new rule given on p. 150: 1 $\overline{3}$ times 12, giving 16. Then squaring 16 to obtain 256, which the scribe then multiplied by 5 $\overline{3}$, giving the answer, 1365 $\overline{3}$ khar. Had he shown the working, or given a hint that the multiplying factor 5 $\overline{3}$ was $\overline{3}$ of the height 8, he would have helped his readers considerably. But finding two-thirds of any numbers at all was mere routine to the Egyptian mathematician, and he no doubt read it off from his two-thirds tables, and thought no more about it. R. C. Archibald, in his bibliography to Chace et al., *The Rhind Mathematical Papyrus*, wrote that Schack-Schackenburg (1899) was the first to explain this problem.